Methods and models of expert fuzzy information processing based on complete orthogonal semantic spaces

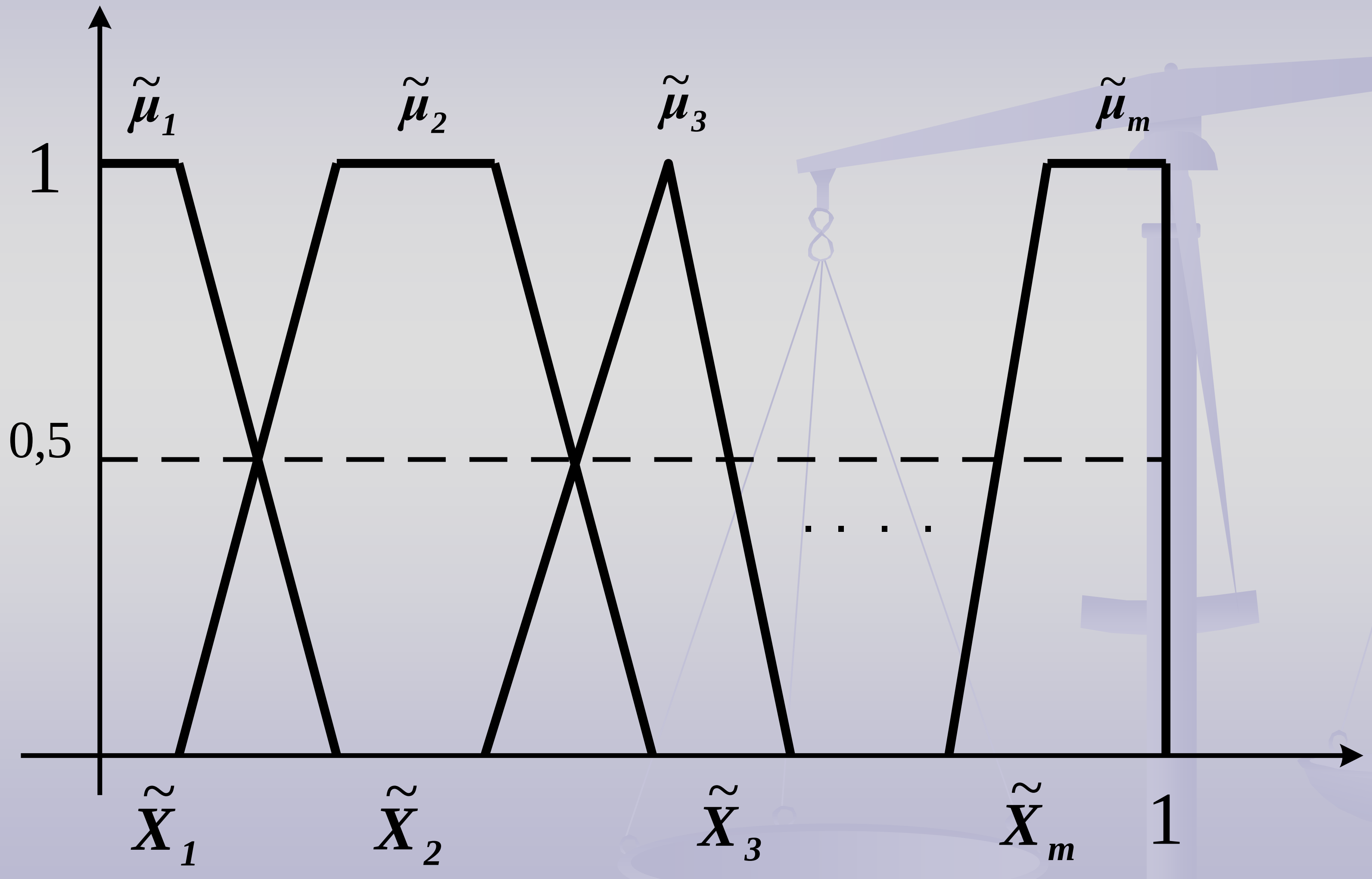
Olga Poleshchuk

Moscow Bauman State Technical University, Russia

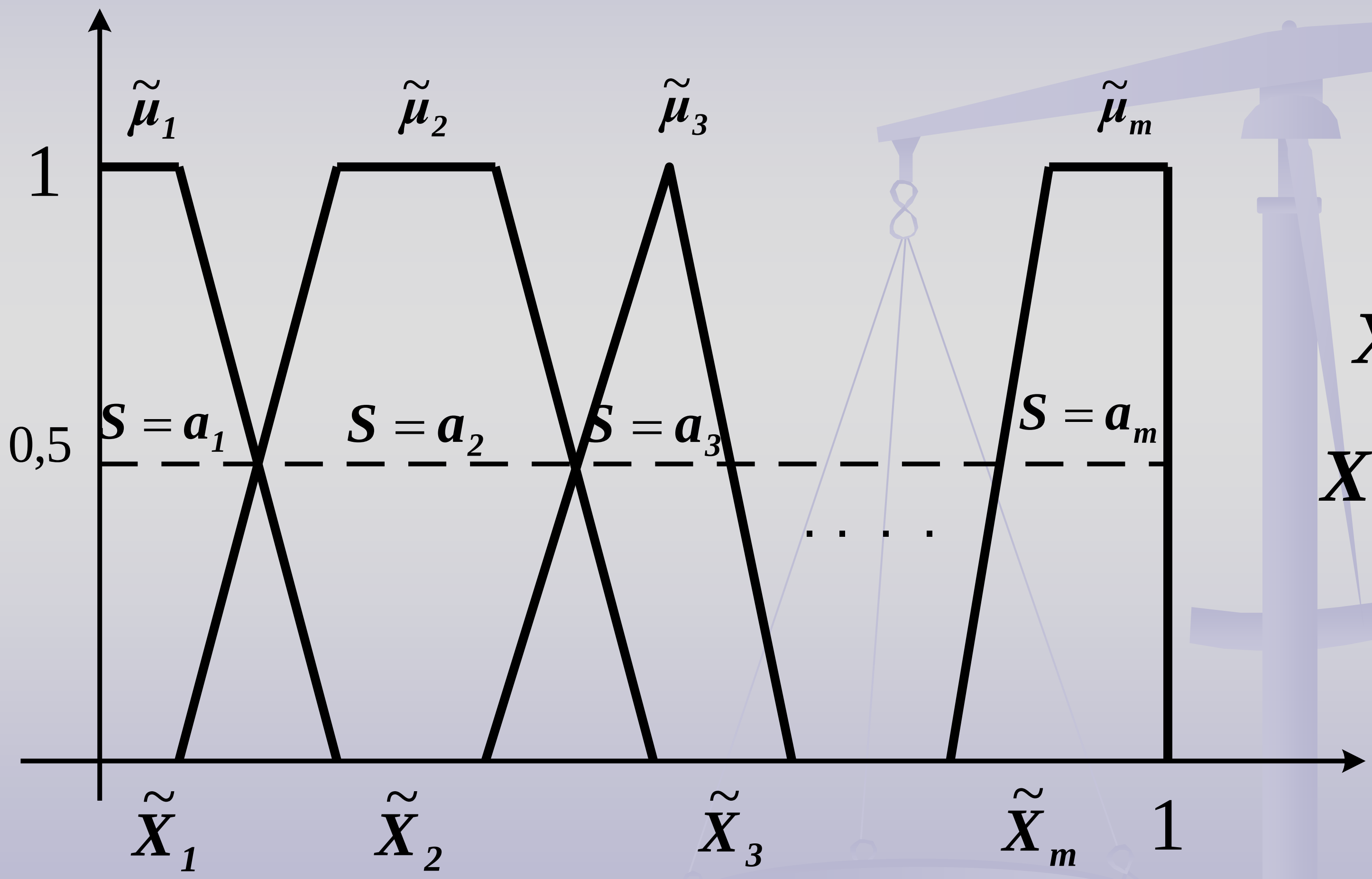
The developed methods

- creating complete orthogonal semantic spaces, as expert assessment models;
- comparative analysis of expert assessment models;
- studying structural composition of models' sets (clusterization);
- creating generalized models of experts' assessments;
- determining rating points of objects and groups of objects in the frame of several characteristics;
- making multiple hybrid fuzzy least-squares regressions (linear and nonlinear) based on the weighted intervals.

Complete orthogonal semantic space



The method for making a complete orthogonal semantic space



N objects

X with levels

$$X_l\,,\ l = \underline{1,m}$$

$$a_l = \frac{n_l}{N}$$

Comparative analysis of expert assessment models

Distinction index

$$d\left(X_i, X_j\right) = \frac{1}{2}\sum_{l=1}^{m}\left|\mu_{il} - \mu_{jl}\right| dx$$

$$X_i = \left\{\mu_{il}(x)\right\},\ \ X_j = \left\{\mu_{jl}(x)\right\}$$

Similarity index

$$r_{ij} = 1 - d\left(X_i, X_j\right)$$

Fuzzy cluster analysis

$$\boxed{R_p = \left(r_{ij}\right),\ i,j = \overline{1,k}}$$

$$\Downarrow$$

Transitive closure

$$\hat{R}_p = \bigcup_{\alpha} \left\{\alpha\left(\delta_{ij}\right)\right\},\ \delta_{ij} = \begin{cases} 0 \\ 1 \end{cases}$$

Creating generelized models of experts' assessments

$$X = \{f_l\}, X_i = \{\mu_{il}\}, l = \overline{1,m}$$

$$\sigma = \frac{1}{k}\sum_{i=1}^{k} d(X_i, X) \rightarrow \min$$

Fuzzy rating points

N objects, $X_j, \; j=\overline{1,k}$ with levels

$X_{lj}, \; l=\overline{1,m}_j$ and $w_j : \sum_j w_j$

$$X_{lj} \to \tilde{X}_{lj}$$

The nth object gets

$\tilde{X}_1^n$ for X_1

. . . .

$\tilde{X}_k^n$ for X_k

$$\tilde{A}_n = \sum_j w_j \tilde{X}_j^n$$

Normed rating points

$$\tilde{A}_n$$

$$\tilde{B}_1 = \sum_j w_j \tilde{X}_{1j}$$

$$\tilde{B}_m = \sum_j w_j \tilde{X}_{mj}$$

The method of gravity center

$\Longrightarrow$

$$A_n$$

$$B_1$$

$$B_m$$

$$E_n = \frac{A_n - B_1}{B_m - B_1}$$

Qualification levels

$\mathbf{1}th$ object $\tilde{A}_1(\mu_1)$ $\quad$ Y_1

$\mathbf{2}th$ object $\tilde{A}_2(\mu_2)$ $\quad$ Y_2

. . . . $\quad$ ....

Nth object $\tilde{A}_N(\mu_N)$ $\quad$ Y_p

$$Y_l \rightarrow \tilde{Y}_l(v_l),\ l = \overline{1,p}$$

Qualification levels

$$\beta_n^l = \frac{\int_0^1 min(v_l, \mu_n)dx}{\int_0^1 max(v_l, \mu_n)dx}$$

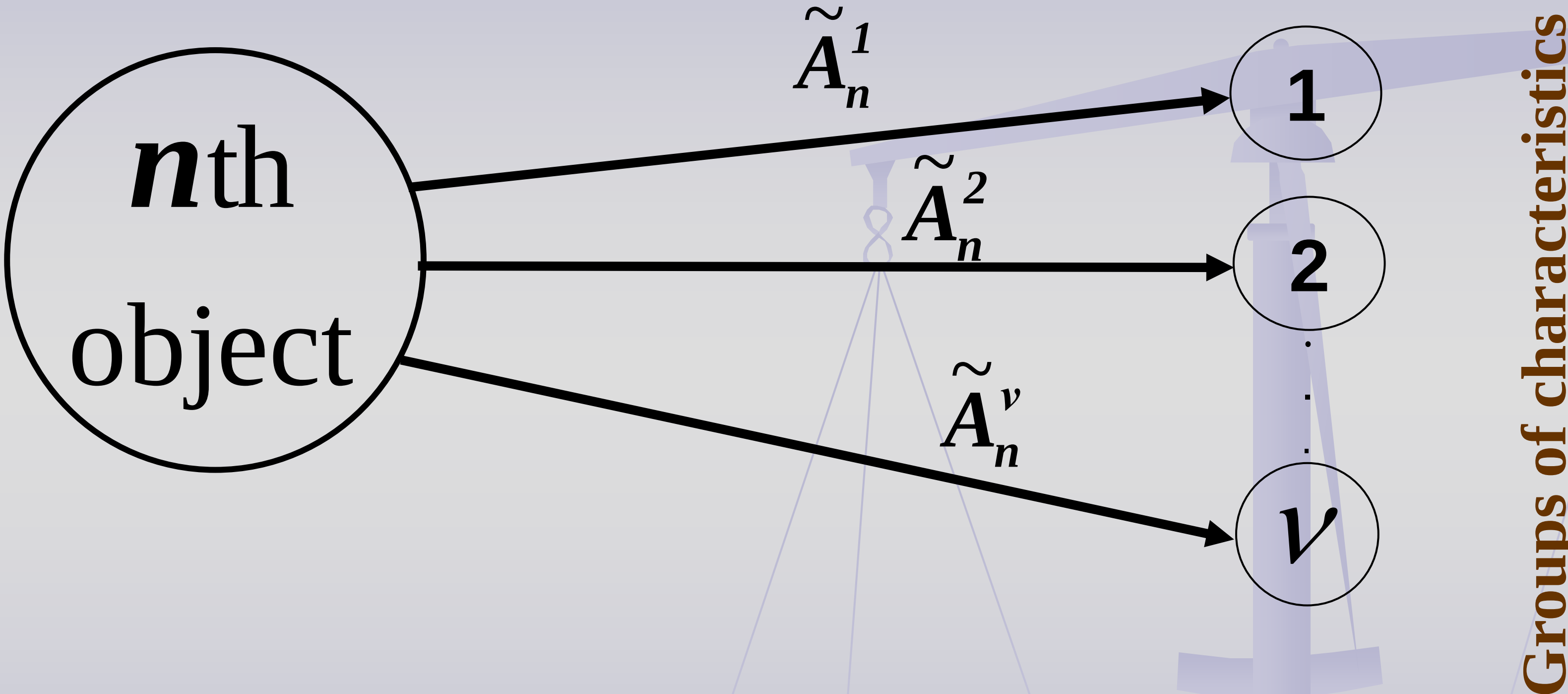
If $\beta_n^j = \max_l \beta_n^l \Rightarrow$ the nth object is assigned Y_j

Clusterization

- C_1 – Not important at all
- C_2 – Rather unimportant
- C_3 – Not very important
- C_4 – Rather important
- C_5 – Important
- C_6 – Very important

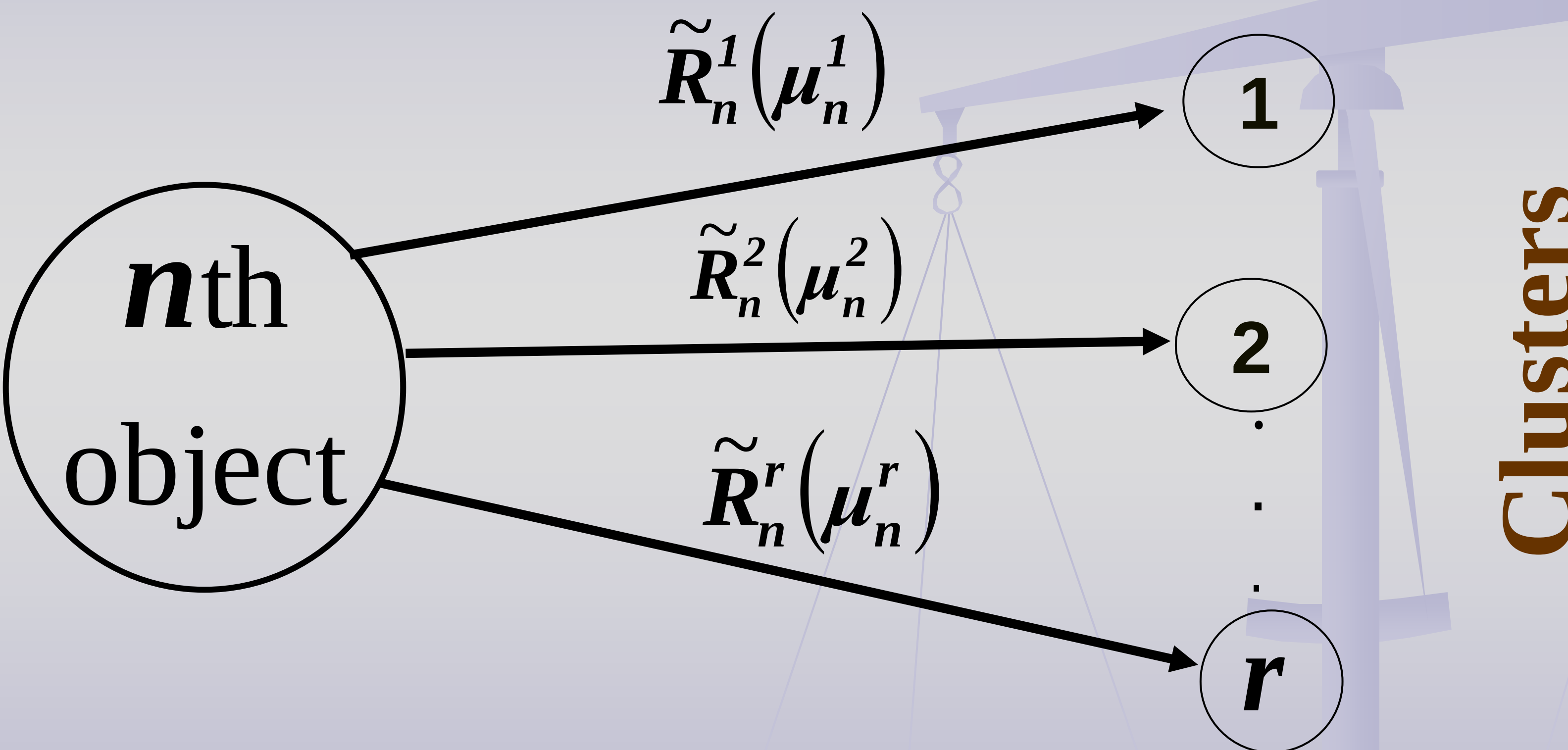
$$C_i \to \tilde{C}_i,\ i = \overline{1,6}$$

Clusterization



$$\tilde{R}_n^i = \tilde{C}_6 \tilde{A}_n^1 \oplus \tilde{C}_2 \tilde{A}_n^2 \oplus \ldots \oplus \tilde{C}_1 \tilde{A}_n^\nu$$

Clusterization



Clusterization

The k -th object is a typical representative of the i-th clusters,

$$(\sup_{n} : \mu_n^i(x) = 1) \in \tilde{R}_k^i$$

Belonging degrees of the other objects:

$$\mu_i(n) = \max_{x} \min(\mu_n^i(x), \mu_k^i(x)),\ n \neq k$$

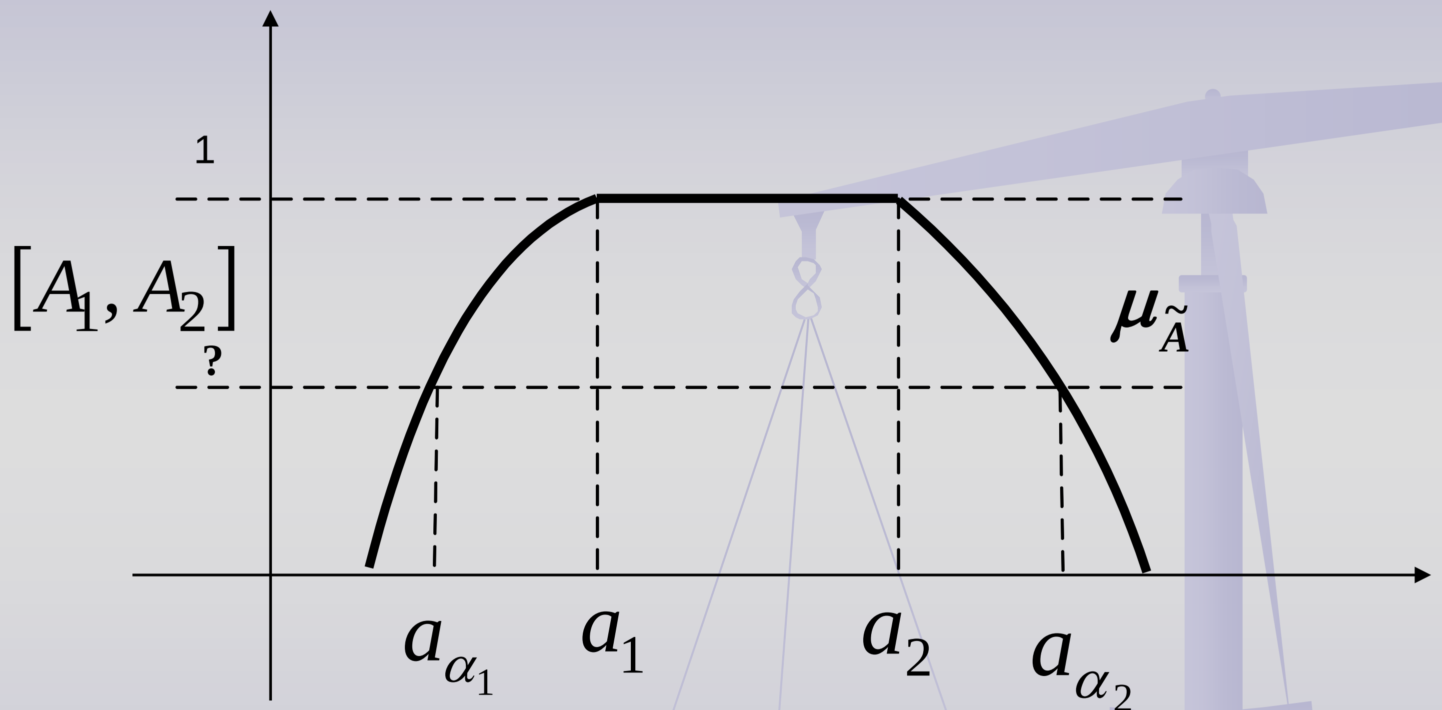
Multiple hybrid fuzzy least-squares regression

$$\tilde{Y} = \tilde{a}_0 + \tilde{a}_1 \tilde{X}_1 + \ldots + \tilde{a}_n \tilde{X}_n$$

$$\tilde{a} \equiv \left(b^j, b_L^j, b_R^j\right), j = \overline{0,m}$$

$$\tilde{X}_j^i \equiv \left(x_1^{ji}, x_2^{ji}, x_L^{ji}, x_R^{ji}\right), \tilde{Y}_i = \left(y_1^i, y_2^i, y_L^i, y_R^i\right)$$

Weighted interval



$$A_1 = \int_0^1 2\alpha\left(\frac{a_1 + a_{\alpha_1}}{2}\right)d\alpha,\ A_2 = \int_0^1 2\alpha\left(\frac{a_2 + a_{\alpha_2}}{2}\right)d\alpha$$

Distance between fuzzy numbers

$$\tilde{A} - \left[A_1, A_2\right]; \quad \tilde{B} - \left[B_1, B_2\right];$$

$$f\left(\tilde{A}, \tilde{B}\right) = \sqrt{\left(A_1 - B_1\right)^2 + \left(A_2 - B_2\right)^2}$$

Optimization problem

$$F = \sum_{i=1}^{n} f^{2}\left(\overset{\frown}{Y}_i, \tilde{Y}_i\right) \to \min,$$

$$b_L^j \geq 0, b_R^j \geq 0, j = \overline{0, m}$$

Conclusions

- **Methods of making complete orthogonal semantic spaces, as expert assessment models.**
- **Methods of comparative analysis of expert assessment models.**
- **Method of studying structural composition of models' sets (clusterization)**
- **Methods of making generalized models of experts' assessments.**
- **Methods of determining rating points of objects and groups of objects in the frame of several characteristics.**
- **Methods of making multiple hybrid fuzzy least-squares regressions (linear and nonlinear) based on weighted intervals.**